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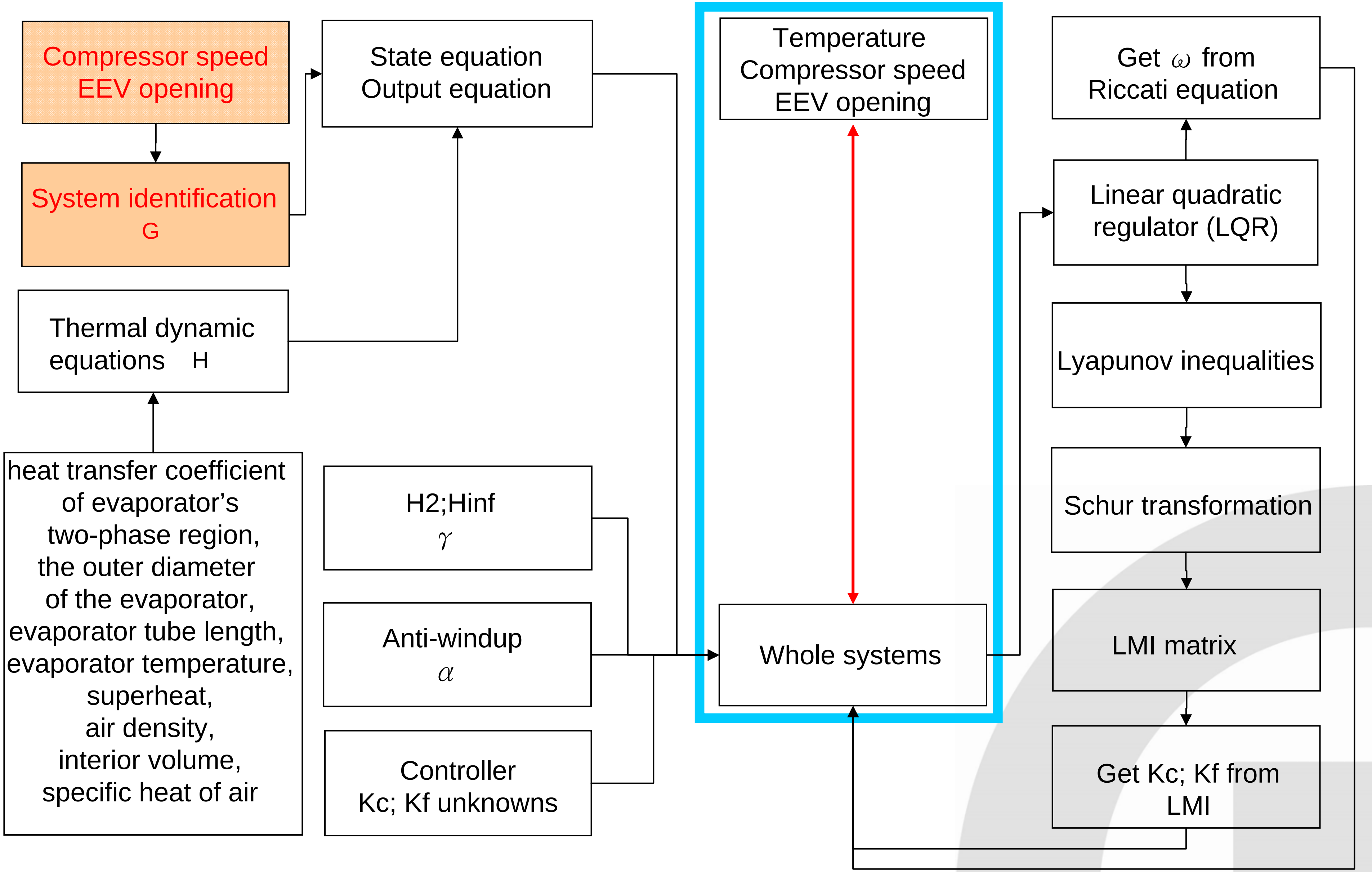
The H_2 / H_∞ Control of the Inverter-fed Air Conditioner Temperature by Linear Matrix Inequalities Based on Linear Quadratic Regulator

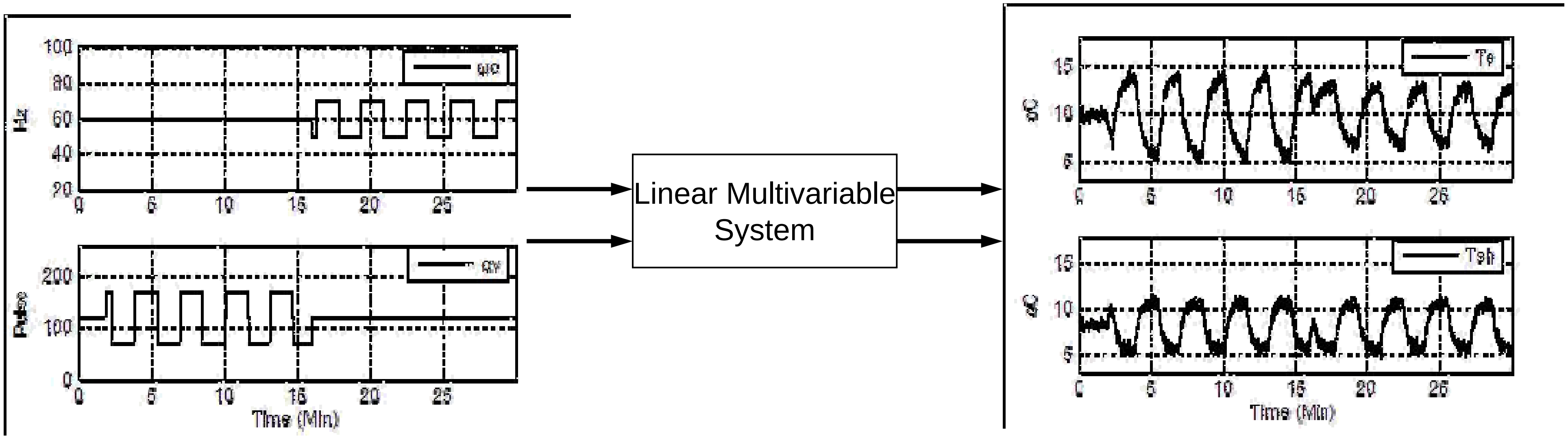
Yaubin Yang (Nov 5~6/2009, Fri~Sat)

Procedure



Compressor speed
EEV opening
System identification
G
Thermal dynamic
equations H
heat transfer coefficient of evaporator's two-phase region, the outer diameter of the evaporator, evaporator tube length, evaporator temperature, superheat, air density, interior volume, specific heat of air
State equation
Output equation
H2;Hinf
γ
Anti-windup
α
Controller
Kc; Kf unknowns
Temperature
Compressor speed
EEV opening
Whole systems
Get ω from
Riccati equation
Linear quadratic regulator (LQR)
Lyapunov inequalities
Schur transformation
LMI matrix
Get Kc; Kf from LMI

System identification G

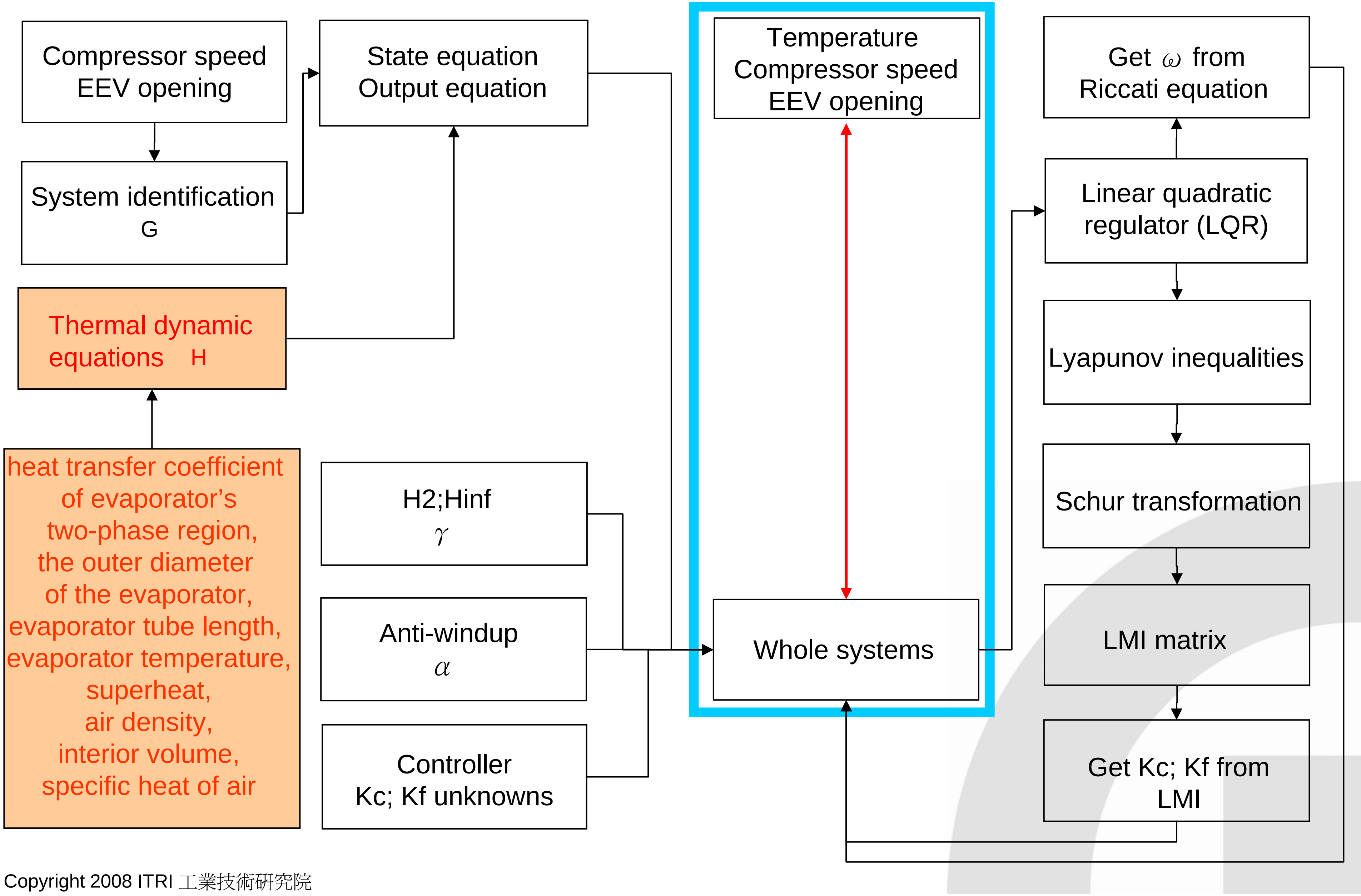


For linear multivariable system, the transfer function is

$$\begin{bmatrix} \overline{T}_e(s) \\ \overline{T}_{sh}(s) \end{bmatrix} = \begin{bmatrix} \overline{G}_{11}(s) & \overline{G}_{12}(s) \\ \overline{G}_{21}(s) & \overline{G}_{22}(s) \end{bmatrix} \begin{bmatrix} \overline{\omega}_c(s) \\ \overline{\alpha}_v(s) \end{bmatrix}$$

$\overline{T}_e(s), \overline{T}_{sh}(s), \overline{G}_{ij}(s), \overline{\omega}_c(s), \overline{\alpha}_v(s)$ are all matrice

Procedure



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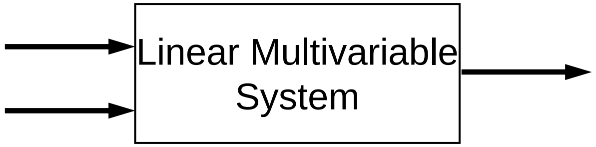
Transfer function H



$$C\frac{dT_r}{dt} = v - \dot{Q}$$

$$\dot{Q} = \alpha_{eo}\pi D_{eo}L_{el}(T_r - T_{ew1}) + \alpha_{eo}\pi D_{eo}(L_e - L_{el})(T_r - T_{ew2})$$

$$\left[\overline{T}_r(s)\right] = \left[\overline{H}_{11}(s) \quad \overline{H}_{12}(s)\right]\begin{bmatrix}\overline{T}_e(s)\\ \overline{T}_{sh}(s)\end{bmatrix}$$

$\overline{T}_r(s), \overline{T}_e(s), \overline{H}_{ij}(s), \overline{T}_{sh}$(s) are all matrice

Where T_r :indoor temperature

C :specific heat

v :heat source

$\dot{Q}$:heat is absorbed by indoor air-conditioner

α_{eo} :thermal conductivity

D_{eo} :evaporator outer diameter

L_e :tube length

L_{el} :two-phase zone length,

T_{ew1} :two-phase region wall temperature

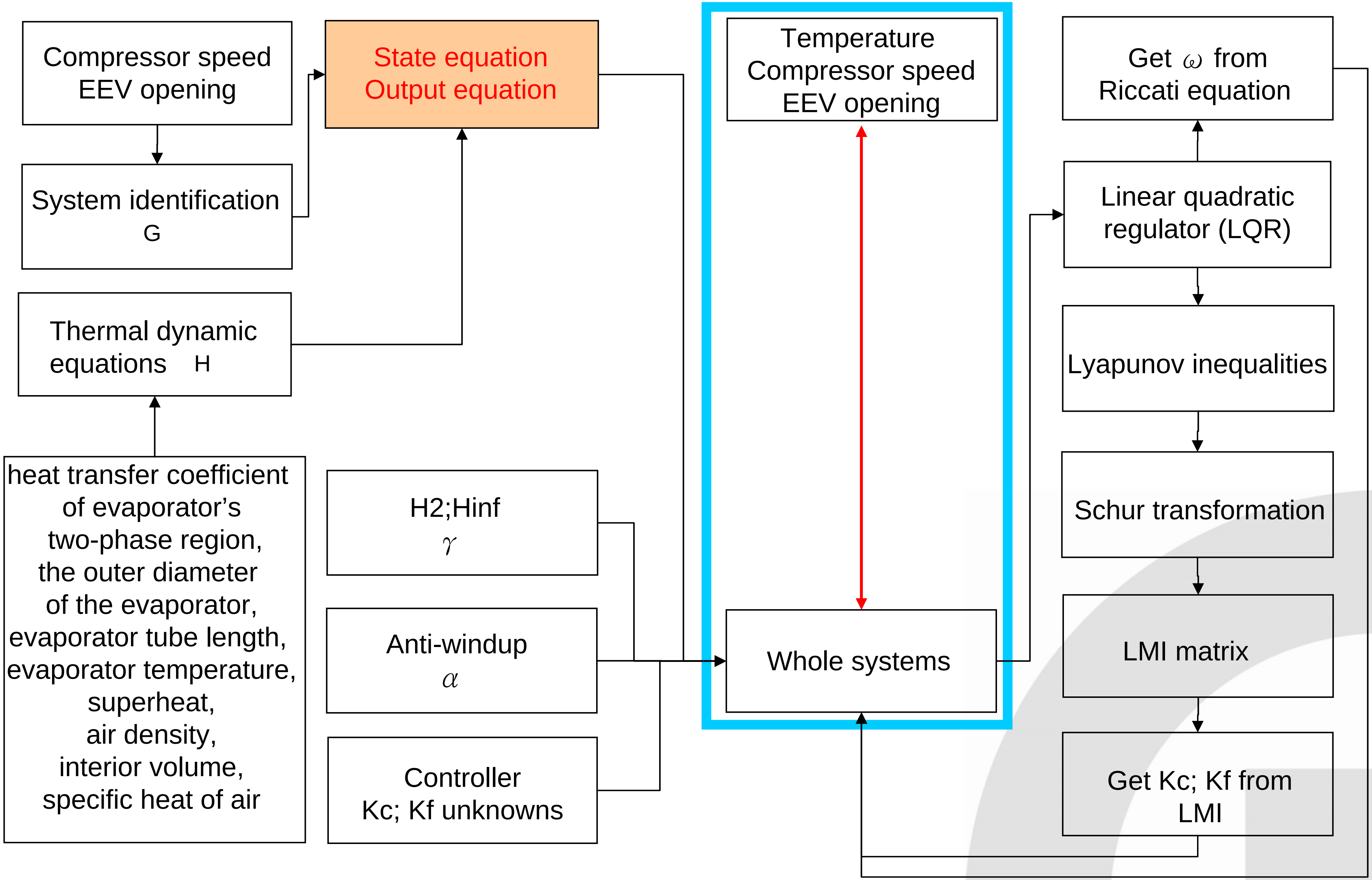
T_{ew2} :superheat zone wall temperature

$$K = \alpha_{eo}\pi D_{eo}$$

$$K_1 = K_2 = K \cdot L_e$$

$$K_3 = (L_e - L_{el})$$

Procedure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Governing equations

$$x(k+1) = Ax(k) + B_2 u_s(k) + B_\eta u_\eta(k) + B_1 \omega(k)$$

$$z(k) = E_1 x(k) + E_2 u_s(k) + G\omega(k)$$

$$u_\eta(k) = -\alpha[u_c(k) - u_s(k)]$$

$$u_c(k) = -Kx(k)$$

$x(k)$: state vectors contain evaporator temp and superheat

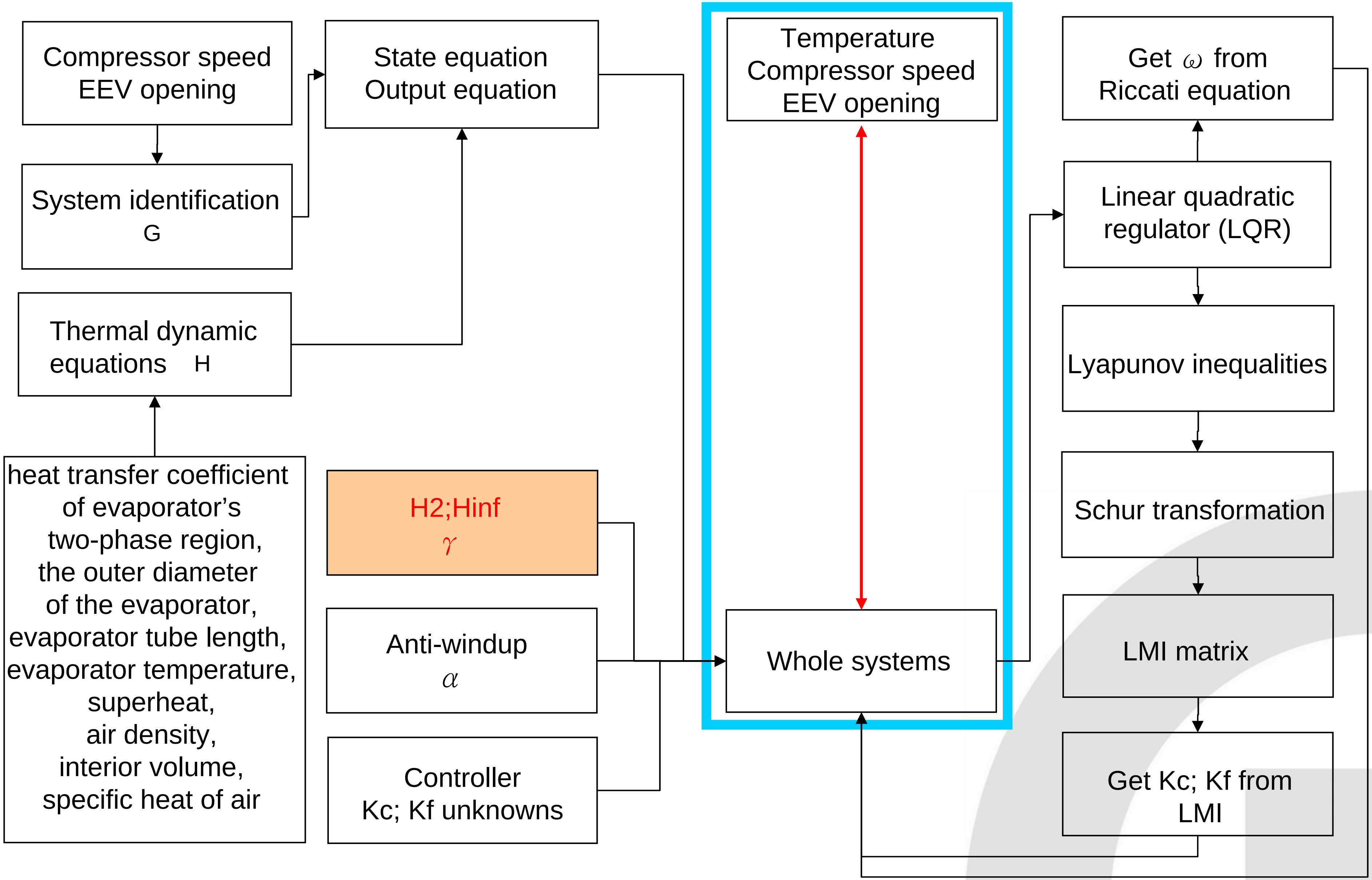
$u_s(k)$: control inputs contain compressor speed, EEV opening, fan speed of indoor and outdoor

$u_\eta(k)$: antiwindup control input if adds integrator

$\omega(k)$: disturbance inputs contain indoor heat source and temp. change

$z(k)$: ouput vectors

Procedure



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$H_2; H_\infty$

$$J_2 = \sum_0^\infty (x^T Q x + u^T R u)$$

Discrete Lyapunov equation

$$APA^T - P - J_2 < 0$$

$$J_\infty = \sup_{\omega \in R} \frac{\|z\|_2^2}{\|\omega\|_2^2} = \sup_{\omega \in R} \frac{\sum z^T z}{\sum \omega^T \omega} \le \gamma^2$$

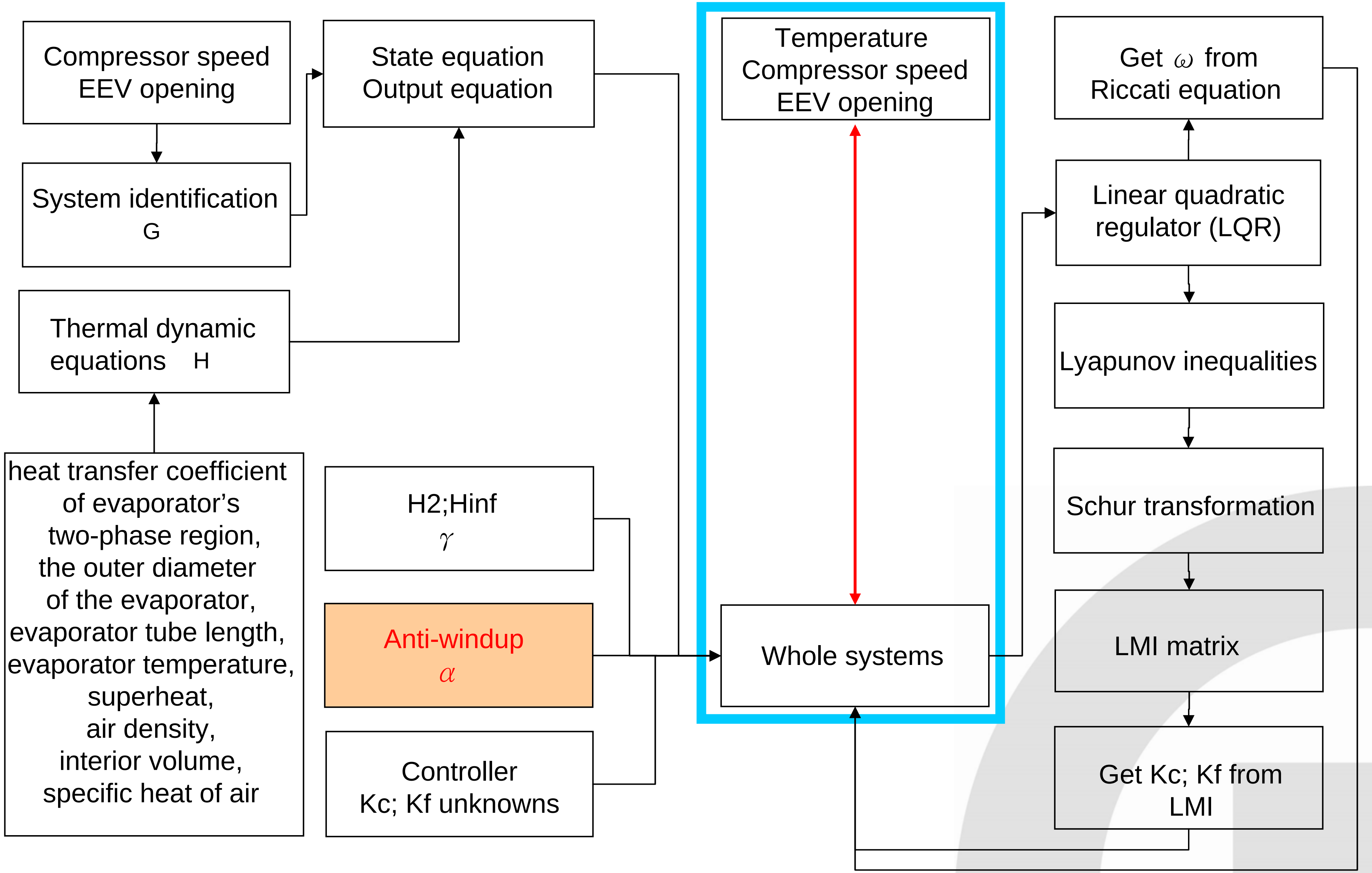
$$\sum_0^\infty (\gamma^2 \omega^T \omega - z^T z) > 0$$

Discrete Riccati equation

$$APA^T - P - J_\infty < 0$$

get ω from Linear quadratic regulator

Procedure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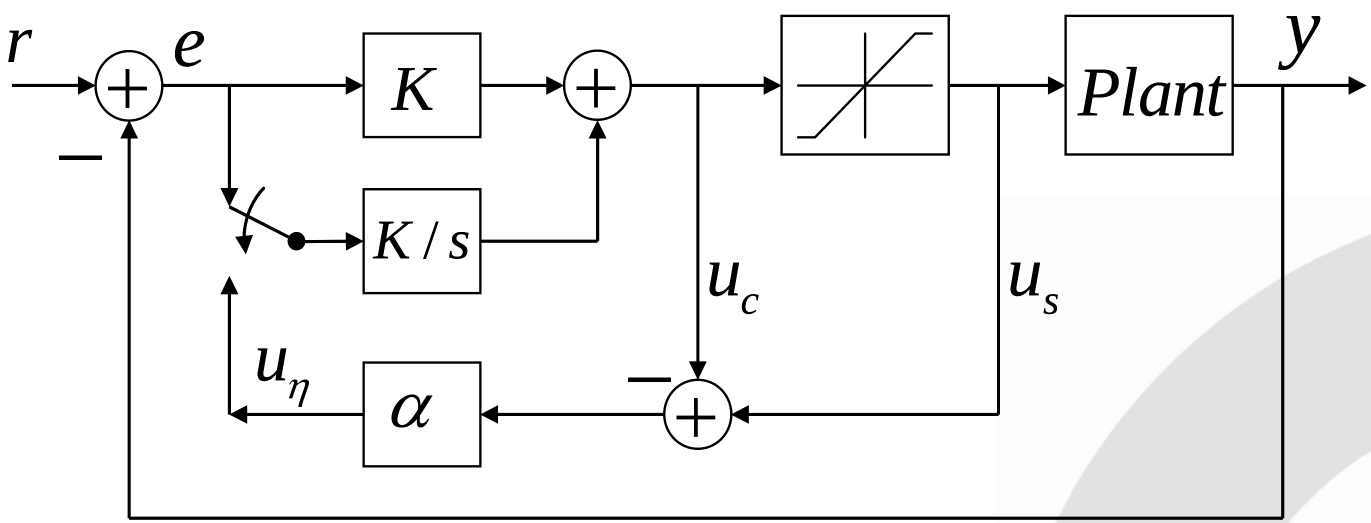
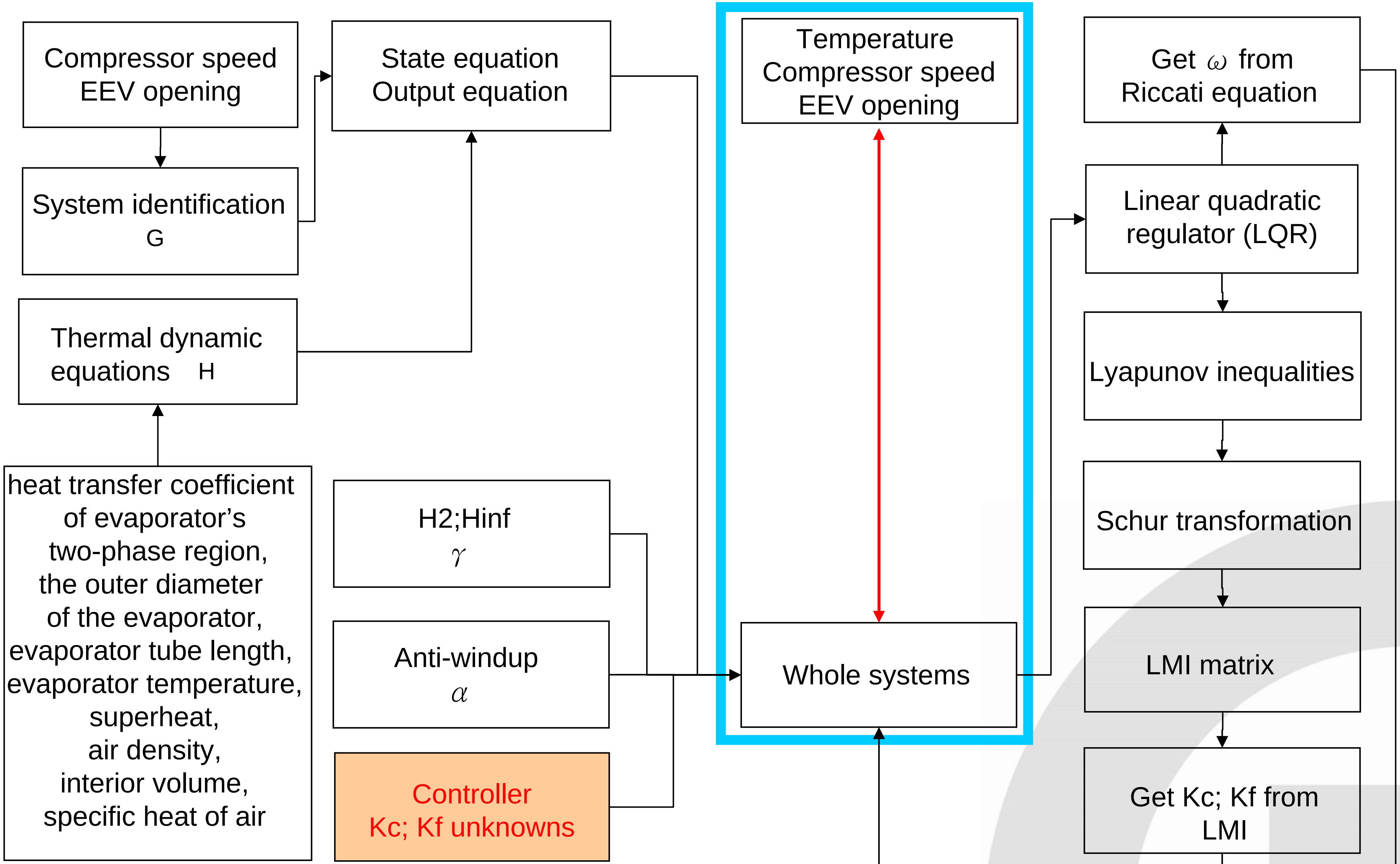
Anti windup

$$u_s = \begin{cases} u_{\min} & \text{for} \quad u_c < u_{\min} \\ u_c & \text{for} \quad u_{\min} < u_c \le u_{\max} \\ u_{\max} & \text{for} \quad u_c > u_{\max} \end{cases}$$



Hodel , A. S., and Hall, C. E., 2001,
Variable-Structure PID control to Prevent Integrator Windup,
IEEE Transactions on Industrial Electronics, vol. 48, no. 2, pp. 442-451.

Procedure



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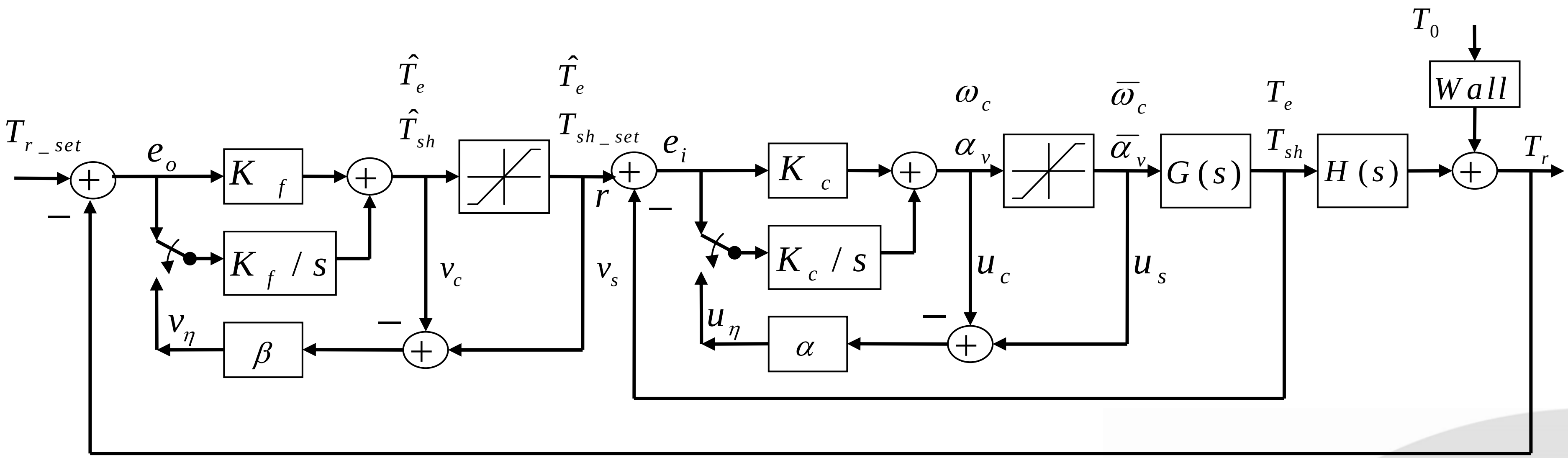
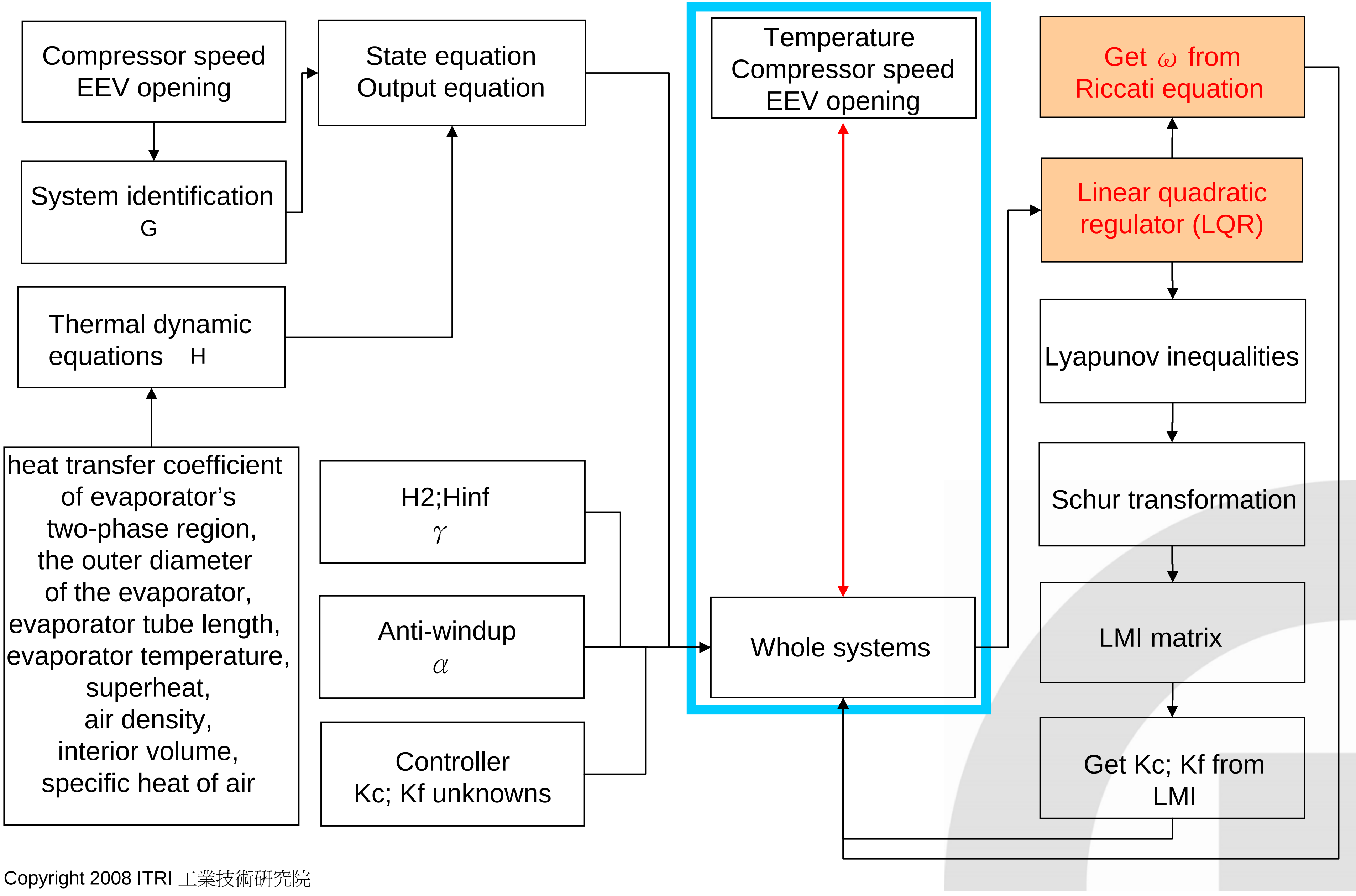
Flow chart



T_0
$Wall$
T_{r_set}
e_o
K_f
K_f / s
$\hat{T}_e$
$\hat{T}_{sh}$
$\hat{T}_e$
T_{sh_set}
r
e_i
K_c
K_c / s
ω_c
α_v
$\bar{\omega}_c$
$\bar{\alpha}_v$
$G(s)$
T_e
T_{sh}
$H(s)$
T_r
v_c
v_s
v_η
β
u_c
u_s
u_η
α

Procedure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Linear quadratic regulator

- For a discrete-time linear system described by

$$x_{k+1} = Ax_k + Bu_k$$

- with a performance index defined as

$$J = \sum_{k=0}^{\infty}(x_k^T Q x_k + u_k^T R u_k)$$

- the optimal control sequence minimizing the performance index is given by

$$u_k = -Kx_k$$

- Where

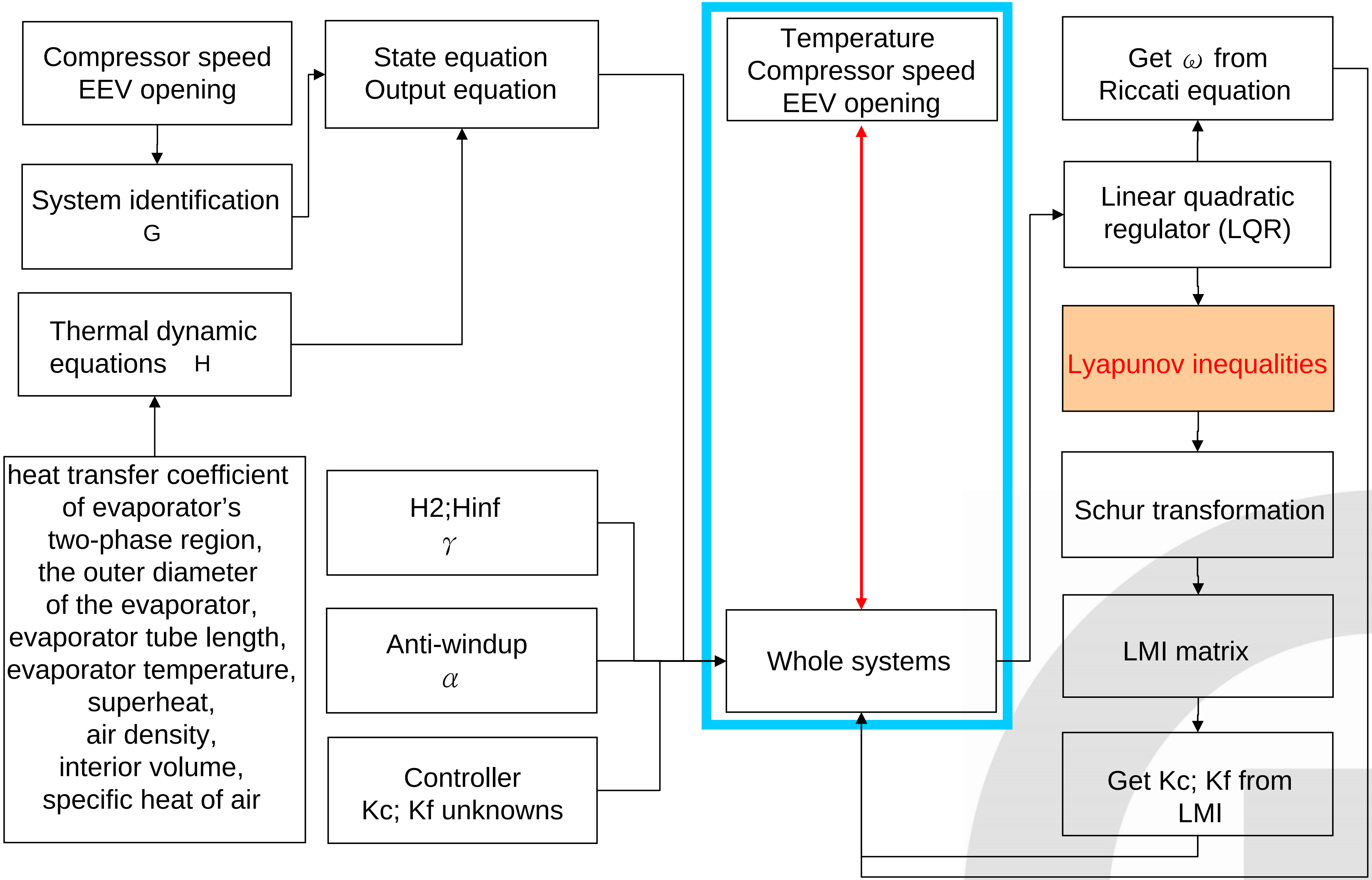
$$K = (R + B^T PB)^{-1} B^T PA$$

- and *P* is the solution to the discrete time algebraic Riccati equation (DARE)

$$P = Q + A^T (P - PB(R + B^T PB)^{-1} B^T P)A$$

From Wikipedia

Procedure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Lyapunov stability

- In the Lyapunov stability theory, the system is said to be asymptotic stable if there exists a symmetric matrix $P = P^T$ in governing equation.
- The function V is a Lyapunov function if $\dot{V}(x)$ is negative semi-definite in U: $\dot{V}(x) \leq 0$.
- The existence of a Lyapunov function is sufficient to prove stability (in the sense of Lyapunov) in the region U.
- If $\dot{V}(x)$ is negative definite, the equilibrium is asymptotically stable.

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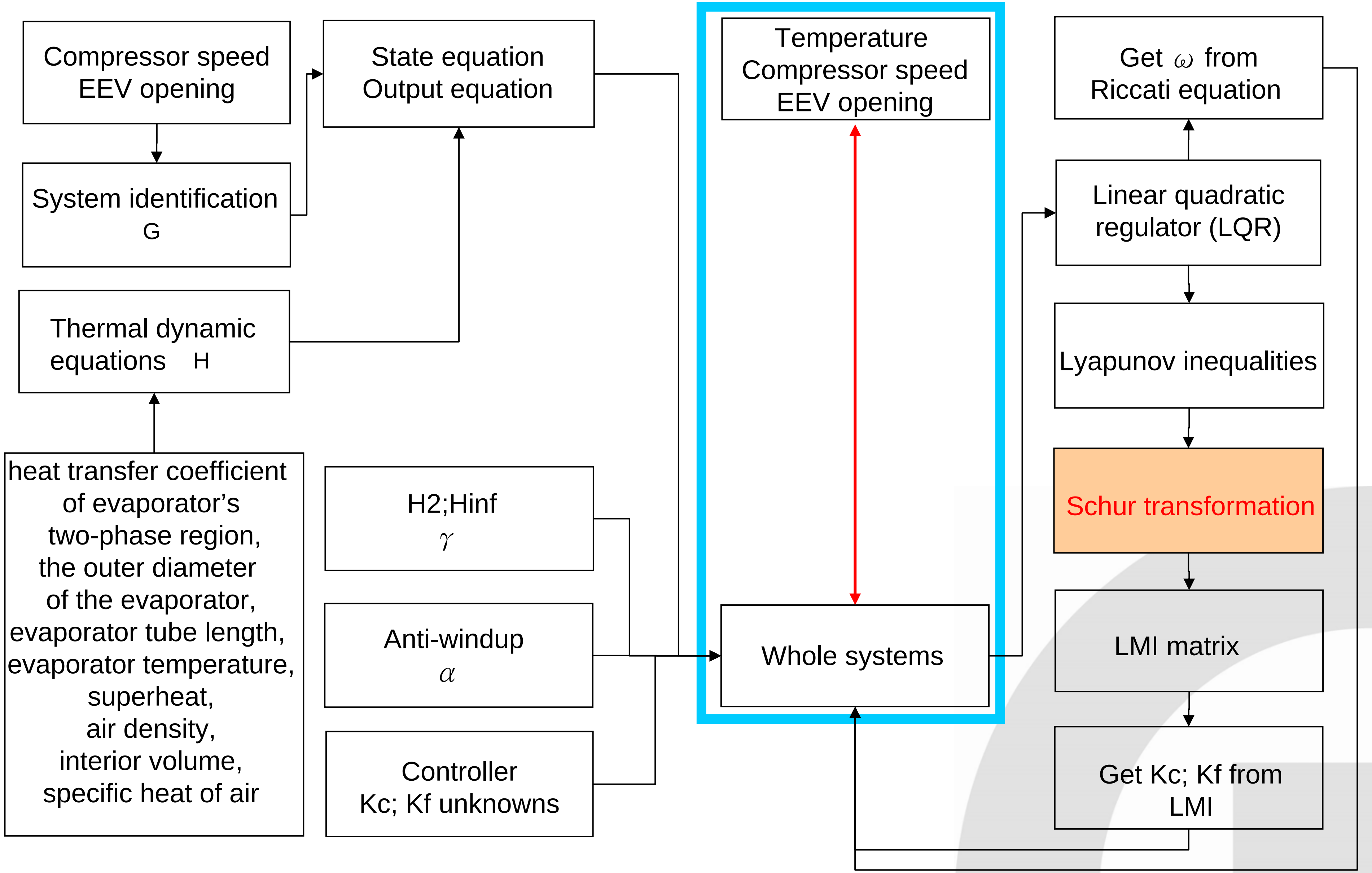
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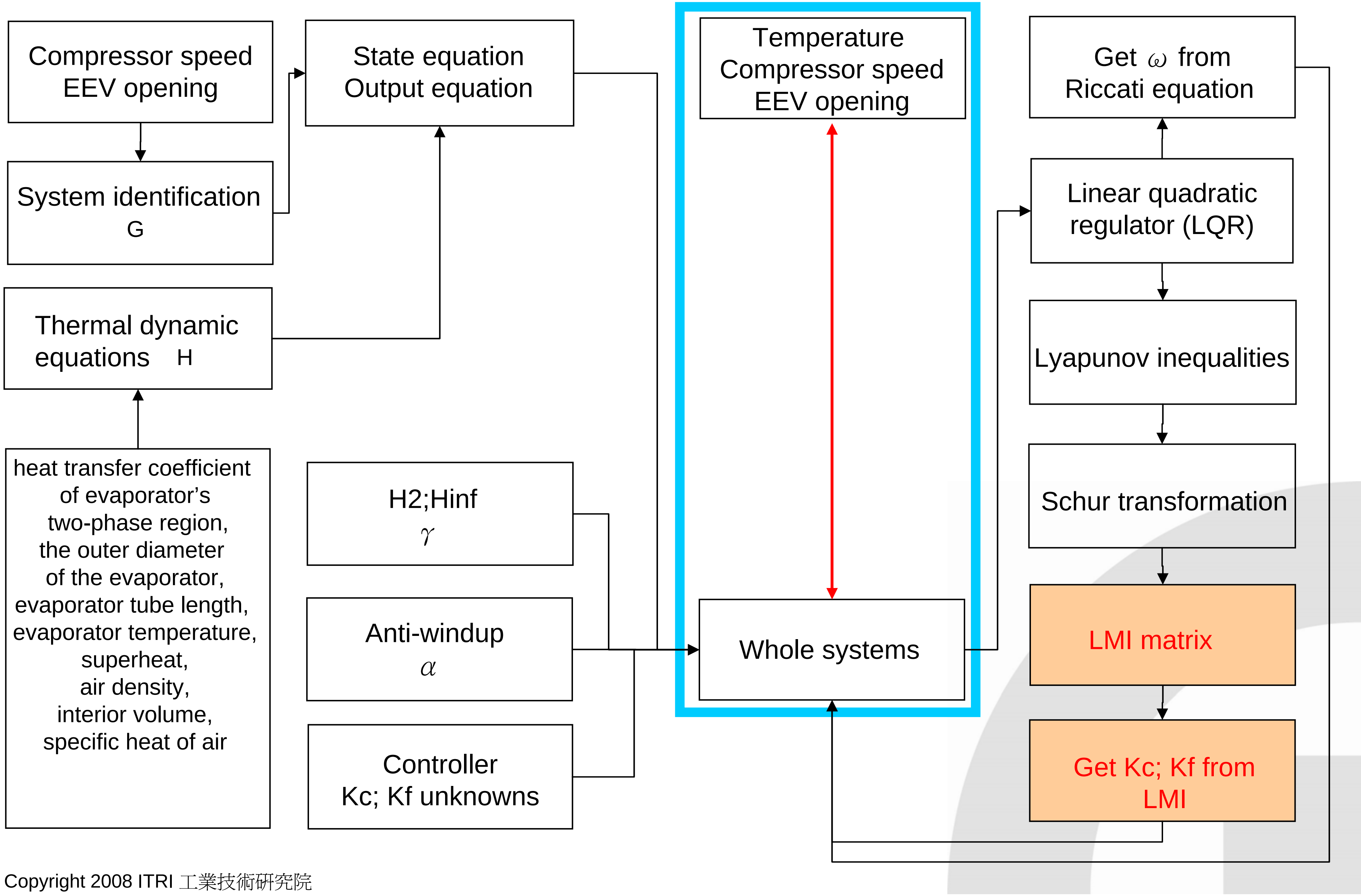
Schur complement:

$$R > 0, Q - SR^{-1}S^T > 0 \Leftrightarrow M = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

inverse matrix expression:

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$$

Procedure



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H_∞ LMI

There exists $Y = Y^T$ such that H_∞ norm LMI in linear operation

$$\begin{bmatrix} W & \bar{\bar{B}}^T \\ \bar{\bar{B}} & Y \end{bmatrix} > 0, Y > 0 \qquad \begin{bmatrix} -Y & * & * & * \\ 0 & E_2^T E_2 & * & * \\ \bar{\bar{A}}Y & \bar{\bar{B}} & -Y & * \\ E_1 Y + E_2 KY & 0 & 0 & -I \end{bmatrix} < 0$$

There exists $Y = Y^T$ such that H_∞ norm LMI in saturation operation

$$\begin{bmatrix} W & \bar{\bar{B}}^T \\ \bar{\bar{B}} & Y \end{bmatrix} > 0, Y > 0 \qquad \begin{bmatrix} -Y & * & * & * & * \\ 0 & E_2^T E_2 & * & * & * \\ 0 & 0 & 0 & * & * \\ \bar{\bar{A}}Y & \bar{\bar{B}} & -\alpha B_\eta & -Y & * \\ \bar{\bar{E}}Y & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

H_2 LMI

There exists $Y = Y^T$ such that H_2 norm LMI in linear operation

$$\begin{bmatrix} -Y & * & * & * & * \\ 0 & 0 & * & * & * \\ \bar{\bar{A}}Y & \bar{\bar{B}} & -Y & * & * \\ 0 & 0 & B_1^T & -\gamma^2 I & * \\ \bar{\bar{E}}Y & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

There exists $Y = Y^T$ such that H_2 norm LMI in saturation operation

$$\begin{bmatrix} -Y & * & * & * & * & * \\ 0 & E_2^T E_2 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ -\bar{\bar{A}}Y & -\bar{\bar{B}} & B_\eta & -Y & * & * \\ 0 & 0 & 0 & B_1^T & -\gamma^2 I & * \\ \bar{\bar{E}}Y & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

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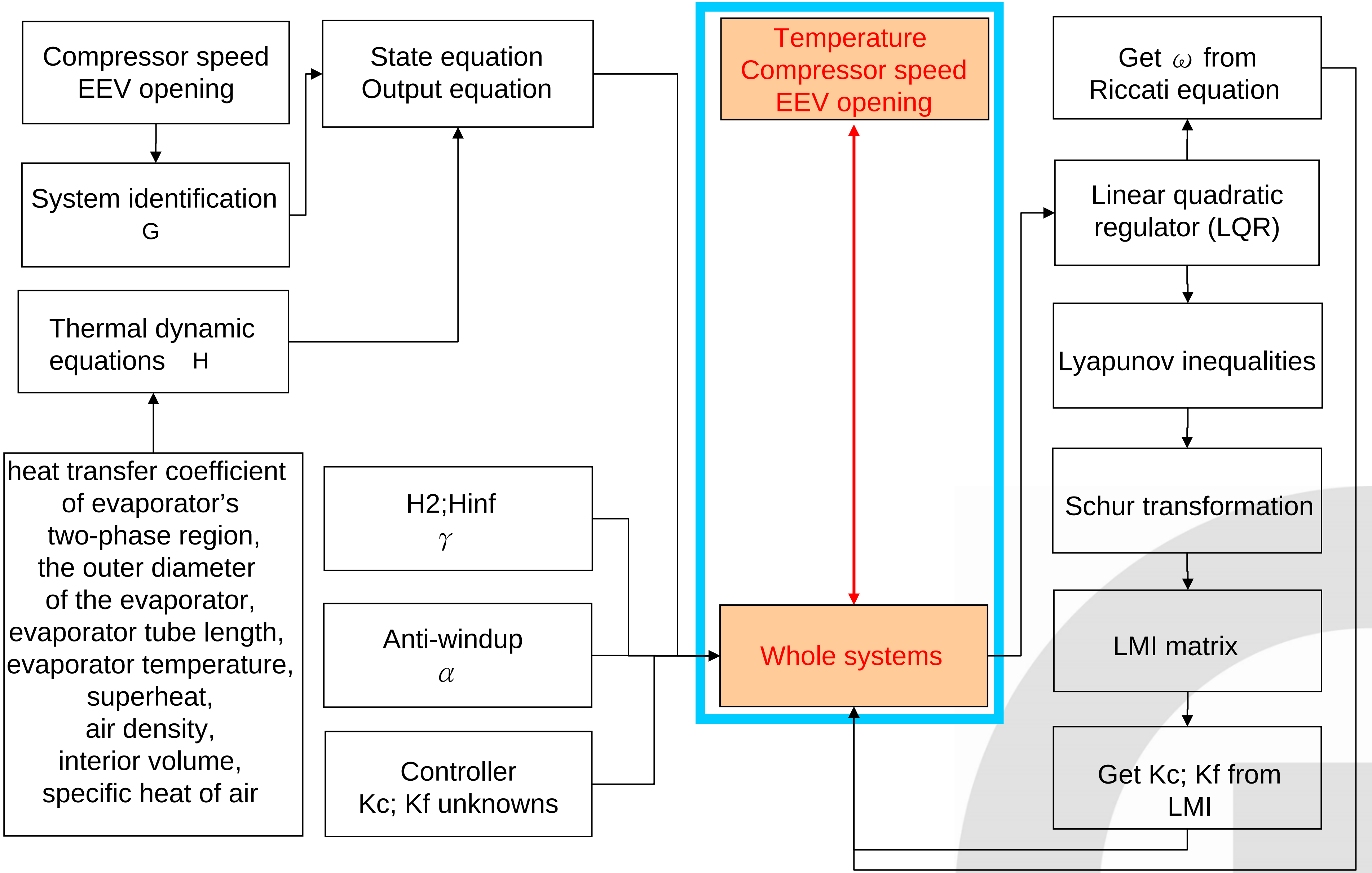
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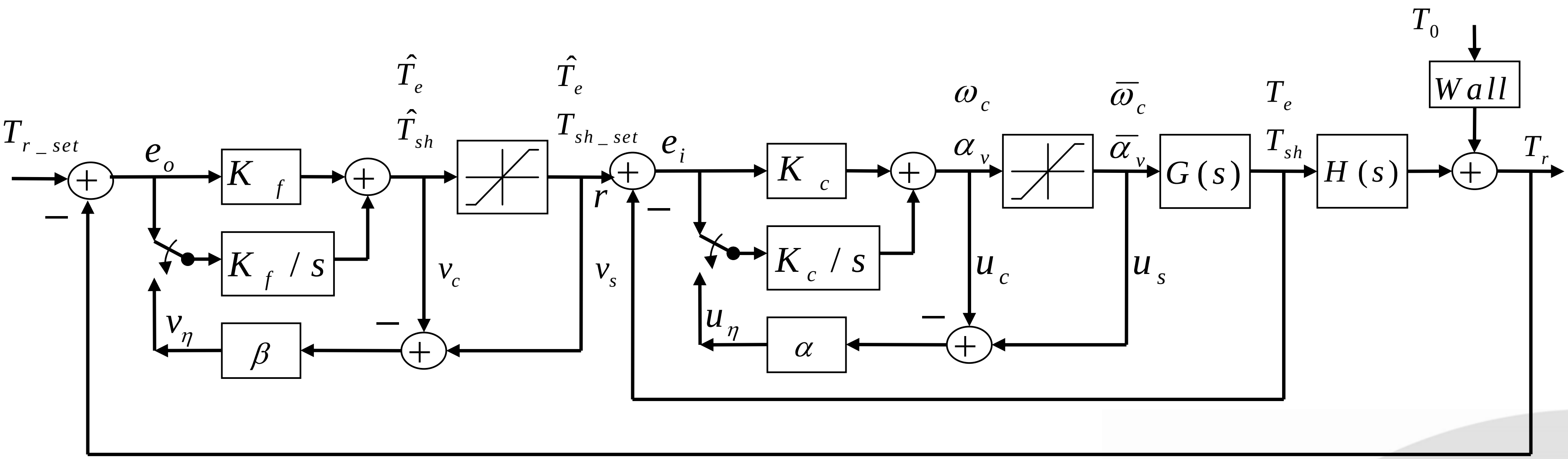

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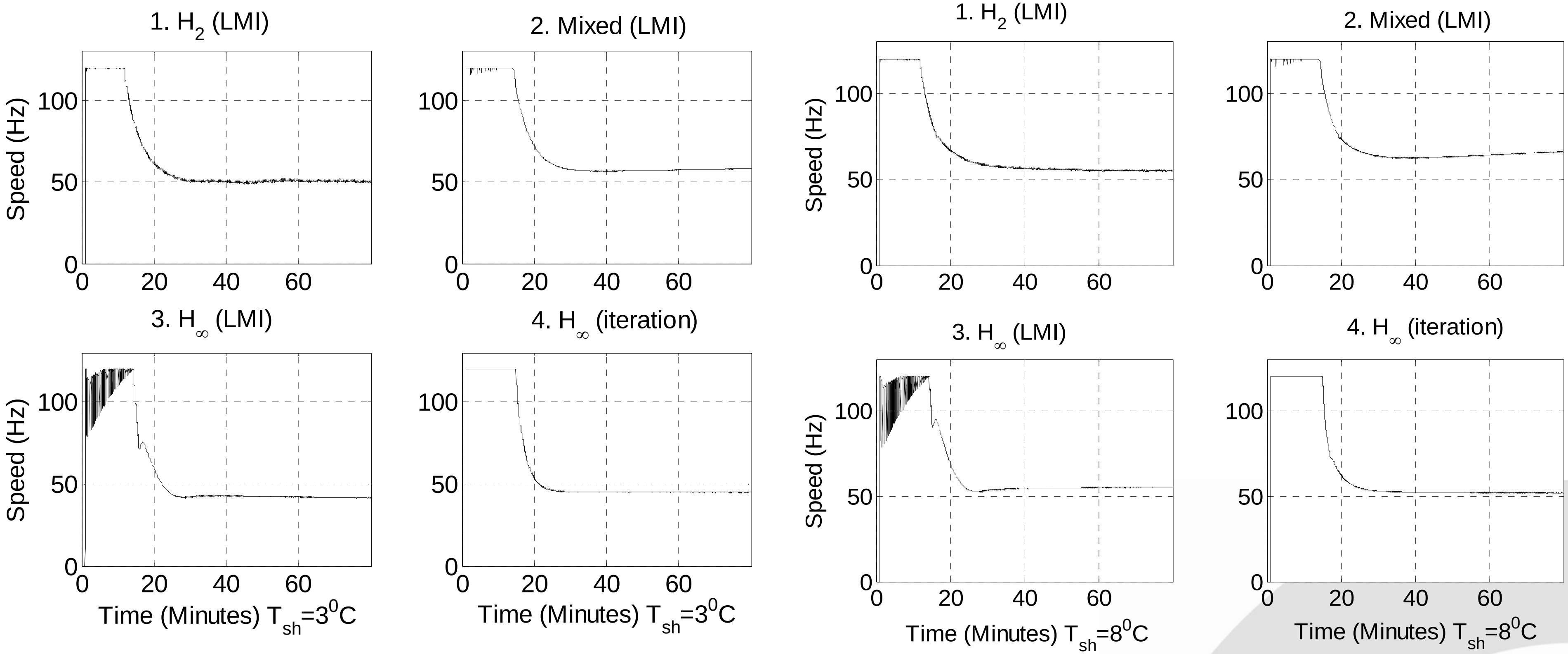
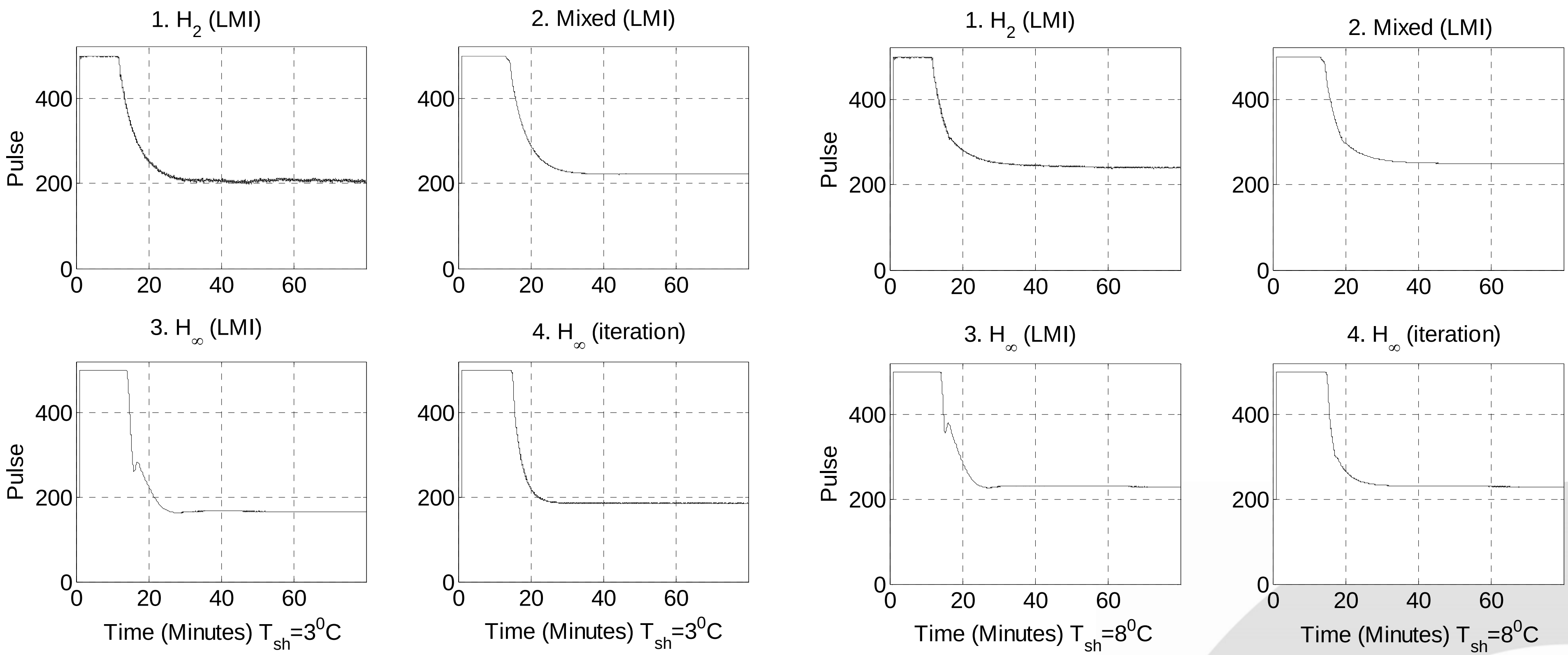
$\hat{T}_{sh}$
$\hat{T}_e$
T_{sh_set}
ω_c
α_v
$\bar{\omega}_c$
$\bar{\alpha}_v$
T_e
T_{sh}
T_{r_set}
e_o
K_f
K_f / s
v_c
v_s
v_η
β
r
e_i
K_c
K_c / s
u_c
u_s
u_η
α
$G(s)$
$H(s)$
T_r

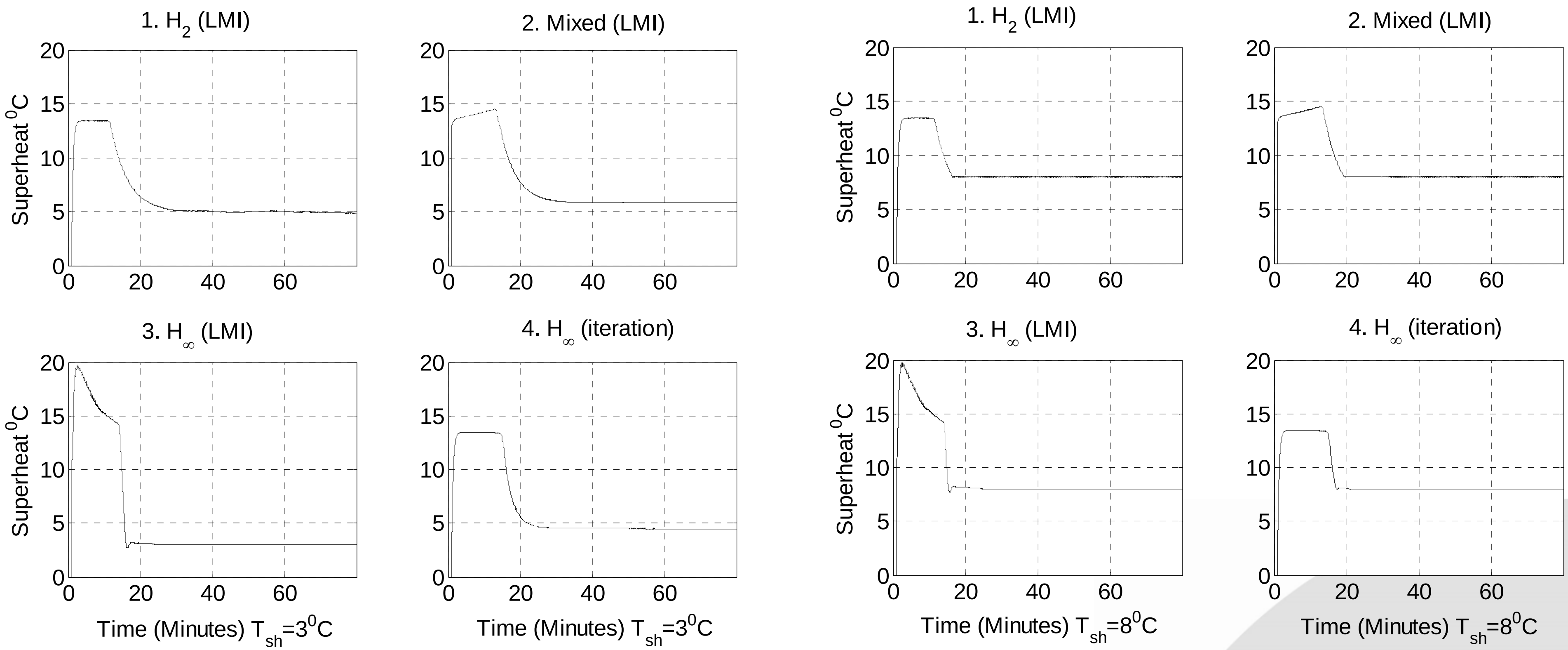
Compressor Speed vs. Time

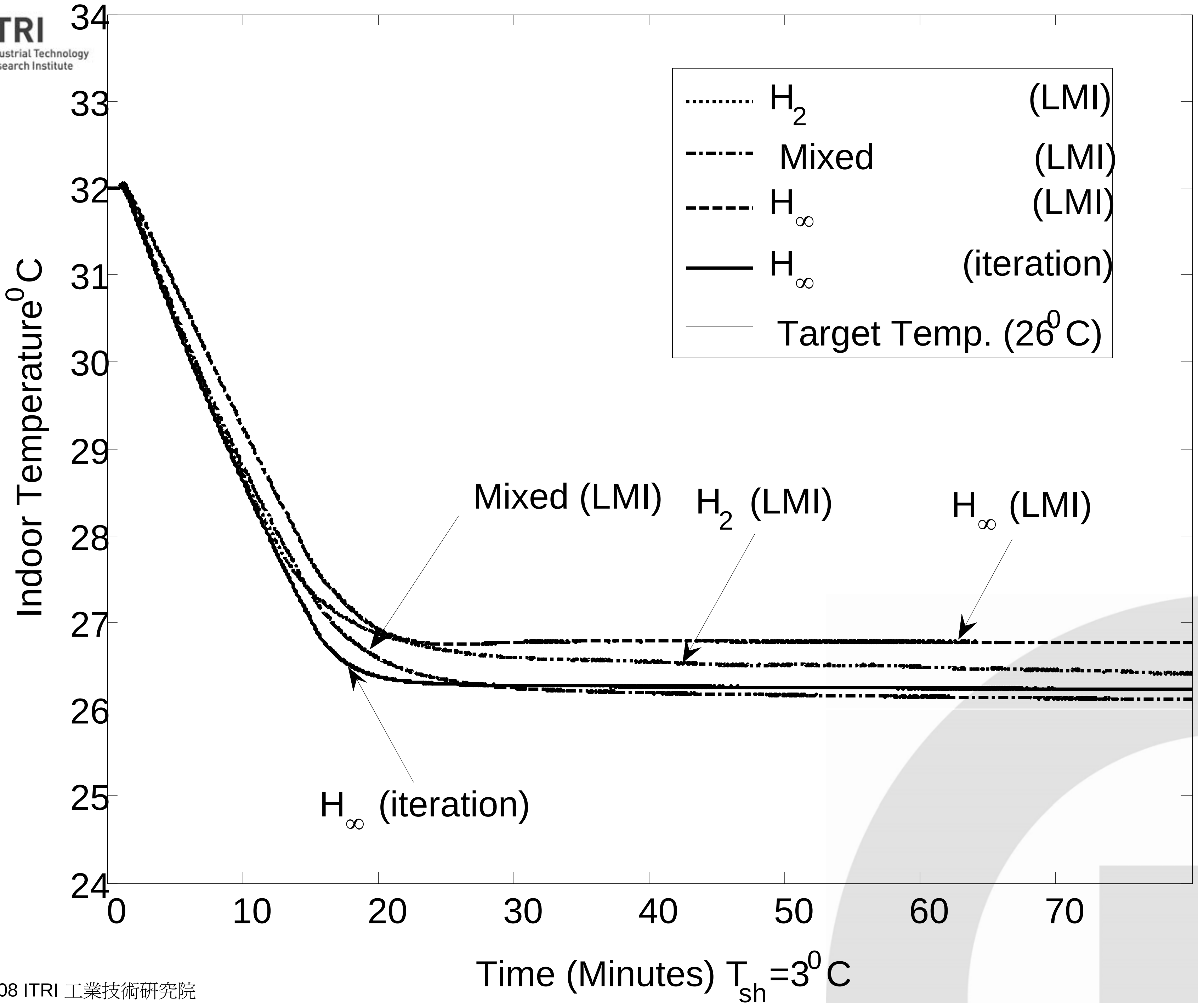


EEV Opening vs. Time

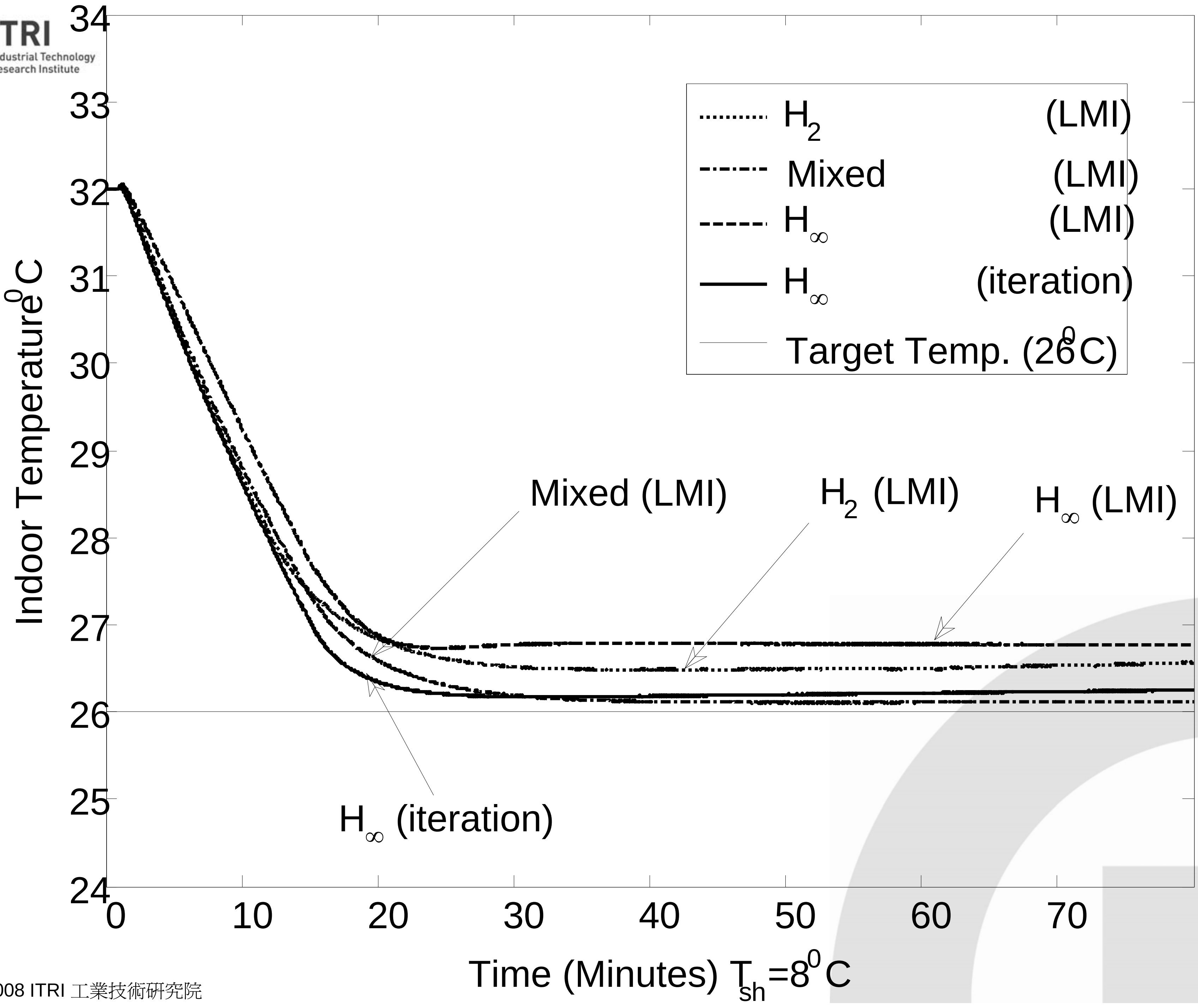


Superheat vs. Time





34
33
32
31
30
29
28
27
26
25
24
0
10
20
30
40
50
60
70
Indoor Temperature ^{0}C
Time (Minutes) $T_{sh}=3^0$C
H_2 (LMI)
Mixed (LMI)
H_∞ (LMI)
H_∞ (iteration)
Target Temp. (26^{0}C)
Mixed (LMI)
H_2 (LMI)
H_∞ (LMI)
H_∞ (iteration)



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Indoor Temperature ^{0}C
H$_2$ (LMI)
Mixed (LMI)
H$_\infty$ (LMI)
H$_\infty$ (iteration)
Target Temp. (26^{0}C)
Mixed (LMI)
H$_2$ (LMI)
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H$_\infty$ (iteration)
0
10
20
30
40
50
60
70
Time (Minutes) $T_{sh}=8^0$C

Thank you for listening